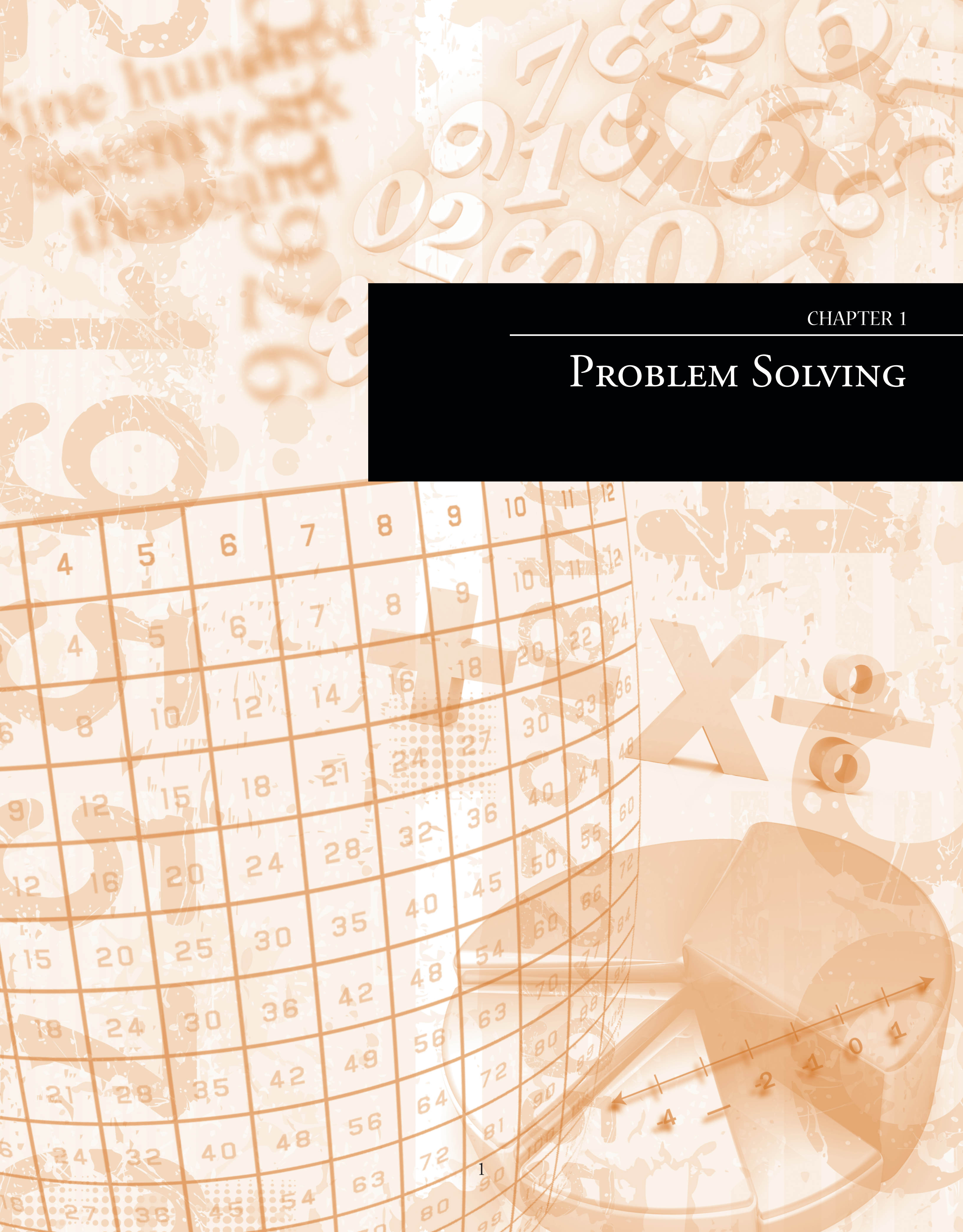
CHAPTER 1

Problem Solving



Assignment Checklist

What you should do:	Where?	When?	☑
Read the Introduction and Objectives	or		☐
Read all sections of the chapter and complete the Practice Problems	or		☐
Complete the assignment and quiz (if applicable) in MathXL	MathXL		☐
Respond to the discussion question(s) for this chapter			☐
Other assignments			☐
Notes:			☐

Introduction

In this chapter, we introduce reasoning, estimating, and problem solving. While many students will argue about the practical applications of many topics studied in an algebra class, most come to an agreement that the mathematics introduced in this chapter has a great everyday impact.

As you move through this chapter, we want you to consider the ways in which this material can impact some of the daily decisions you make. Whether these decisions involve verifying a percentage discount on a bill, estimating a proper tip, or making a major purchase, we hope that you will find the material studied in this chapter useful when making such decisions.

Learning Objectives

When you finish your study of this section, you should be able to

- Define inductive reasoning and deductive reasoning
- Use inductive reasoning to complete numeric sequences
- Use deductive reasoning to prove a conjecture
- Round numbers to a given place value
- Use rounding to estimate when solving word problems
- Solve word problems involving percents

Section 1
Inductive and Deductive Reasoning

Introduction

Inductive reasoning is the process of arriving at a general conclusion based on observations of specific examples. In other words, we look for a pattern in whatever we are studying and then try to draw a conclusion based on that pattern. For example, Prof. Shawver notices that, for the past three semesters, students who always sit in the back row of his classroom fail his course. At the end of the third semester, Prof. Shawver makes a conjecture (a ***hypothesis***, or educated guess) that anyone always sitting in the back row will receive an F in the course. This is an example of inductive reasoning. By observing, over three straight semesters, students always sitting in the back of the classroom (specific examples), Prof. Shawver arrives at the conclusion that any student always sitting there will fail the course.

In the real world, inductive reasoning can assist us when patterns are evident; it can help detectives solve various types of crimes, NFL® teams choose their future star quarterbacks, stockbrokers forecast the performance of stocks, and so on, but ***we can never be certain of conclusions arrived at using inductive reasoning***. In order to disprove a conjecture made by inductive reasoning, we would need to find only one example (called a ***counterexample***) in which the conjecture is false. In Prof. Shawver's class, if one student passes the course after always sitting in the back row, Prof. Shawver's conjecture has been proven false. In the cases in which inductive reasoning is used to solve crimes, choose quarterbacks, and identify strongly performing stocks, many counterexamples exist of innocent people imprisoned, overrated quarterbacks drafted in the first round, and "buy" stocks "tanking" (see Enron), all based on observations of patterns that detectives, teams, and stockbrokers thought they were seeing. That said, in the field of mathematics, inductive reasoning is very useful in identifying patterns and solving problems.

In the following example, we will use inductive reasoning to identify a pattern and then determine the next number in the sequence.

Example 1: Use inductive reasoning to find the next number in the following sequences.

Part A: 3, 7, 12, 18, 25, __?

When you look at this problem, first try to determine if a pattern involving addition or subtraction exists between the terms of the sequence. Notice the pattern that follows:

3 (+ 4), 7 (+ 5), 12 (+ 6), 18 (+ 7), 25

Continuing with the pattern, adding 8 to 25 will give us the solution of 33.

Answer: 33

Part B: 1, 3, 9, 27, __?

Again, look for a simple addition or subtraction pattern between the terms. When this problem is analyzed, no obvious pattern is seen:

1 (+ 2), 3 (+ 6), 9 (+ 16), 27

Since a pattern involving addition or subtraction isn't apparent, try looking for a pattern involving multiplication or division. A multiplication pattern *is* apparent:

1 (• 3), 3 (• 3), 9 (• 3), 27. . .

Continuing with the pattern, multiplying the 27 by 3 will give us the solution of 81.

Answer: 81

Part C: 8, -4, 2, -1, $\frac{1}{2}$, __?

First you may notice that the signs are alternating in this sequence. If the signs are alternating, you should expect some pattern involving multiplication or division. In this example, you could have seen one of two patterns:

Pattern 1: $8\ (\div -2), -4\ (\div -2), 2\ (\div -2)\ -1\ (\div -2), \frac{1}{2}\ (\div -2)$

Pattern 2: $8\ (\bullet -\frac{1}{2}), -4\ (\bullet -\frac{1}{2}), 2\ (\bullet -\frac{1}{2}), -1\ (\bullet -\frac{1}{2}), \frac{1}{2}\ (\bullet -\frac{1}{2})$

In either case, the next number in the pattern will be the fraction $-\frac{1}{4}$.

Answer: $-\frac{1}{4}$

Part D: 1, 1, 2, 3, 5, 8, 13, __?

In this famous problem, you may not see any pattern involving addition, subtraction, multiplication, or division. So what else may the pattern involve? Well, in this case, let's look at the *two* previous terms to see how we might obtain the next number:

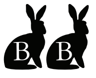
1, 1 (+ 1), 2 (+ 1), 3 (+ 2), 5 (+ 3), 8 (+ 5), 13 (+ 8),

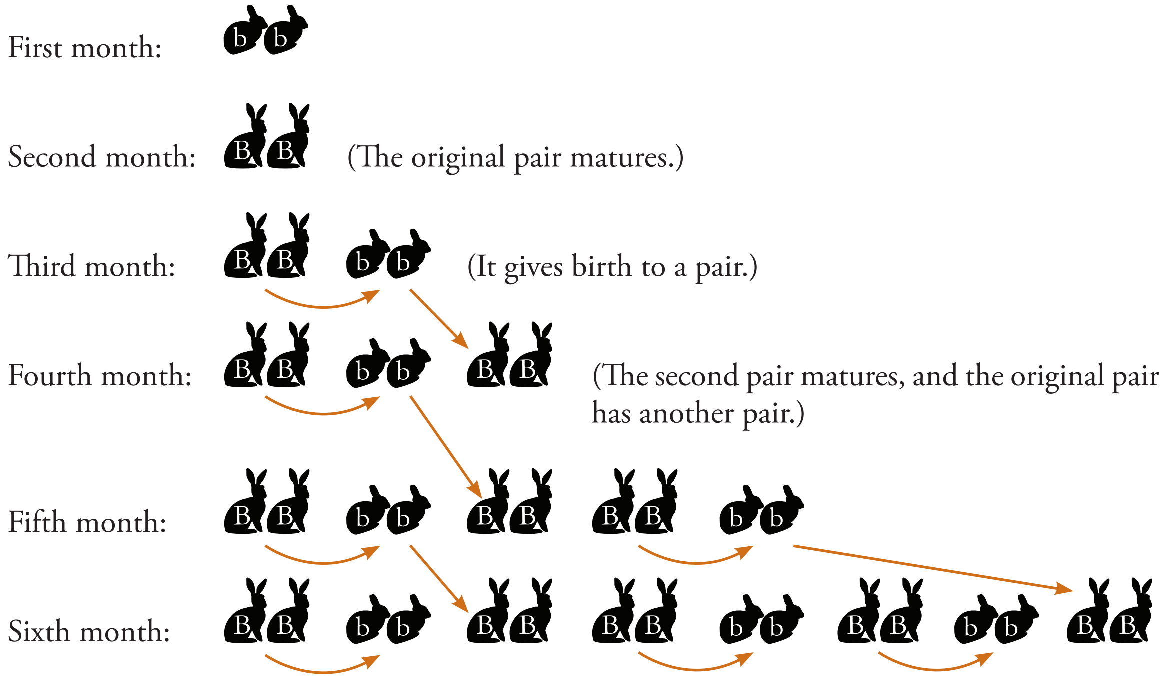
8 + 13 = 21, our answer!

This number sequence is the famous ***Fibonacci sequence***; the next number in the sequence is obtained by adding the two previous numbers.

Answer: 21

The Fibonacci sequence originated as the solution to the following problem. Suppose a farmer has pairs of rabbits. It takes a month for the rabbits to mature. Once mature, a pair of rabbits gives birth to one pair of rabbits (one male and one female) each month. How many pairs of rabbits does the farmer have each month?

Let  represent immature bunnies and  represent mature bunnies.



Count the number of pairs each month: 1, 1, 2, 3, 5, 8—the Fibonacci sequence!

You can find a considerable amount of information online about the Fibonacci sequence. Be careful, though, to sift through the nonsense that has been written about the sequence.

Using Inductive Reasoning to Test a Conjecture

While inductive reasoning is very useful for identifying patterns and finding missing numbers in sequences, it is also useful in testing conjectures about numbers and their properties. Let's look at an example.

Example 2: Use inductive reasoning to test the following conjectures. If the conjecture is false, give a counterexample.

Part A: The sum of two positive odd numbers is always an even number.

To use inductive reasoning, we need to list several examples to see if the conjecture appears to be true. We need to list only one example showing the conjecture doesn't hold true; it will be our counterexample. Thus, a list of pairs of odd numbers, along with their sums, follows.

1 + 1 = 2

1 + 3 = 4

3 + 5 = 8

7 + 9 = 16

9 + 9 = 18

103 + 47 = 150

Answer: While we can't be 100% sure that this conjecture is true, through inductive reasoning and our six examples, we strongly suspect that the conjecture is true.

Part B: If a number is added to itself, the sum is greater than the original number.

Again, let's list some examples to see if this conjecture appears to be true.

1 + 1 = 2

4 + 4 = 8

10 + 10 = 20

So far, it seems as if this conjecture is true, but wait--we didn't consider negative numbers:

-2 + -2 = -4

Answer: Since -4 is smaller than the original number, -2, this counterexample shows that this conjecture is false.

Note: When you are using inductive reasoning to show that a conjecture is true, you should look at many examples. And, to decisively prove that the conjecture is true, you should use deductive reasoning, which we will discuss in the next section. However, in order to show that a conjecture is false, only one counterexample is necessary.

Deductive Reasoning

Deductive reasoning involves creating a chain of conclusions based on a fact or series of facts. The first fact leads to a conclusion, this conclusion leads to a second conclusion, etc. This type of reasoning was made famous by Sherlock Holmes and other fictional detectives. Let's look at some examples to see if we can differentiate between inductive and deductive reasoning.

Example 3: Bill and Jerry's Ice Cream Company created a new flavor called Super Berry. The owners went to 10 different schools to give out 20 free samples to students at each school, seeking feedback on the new flavor. All the students who tasted the new flavor said that it was the best ice cream they had ever tasted. Bill and Jerry concluded that this ice cream would be a great seller for the company. Is this an example of inductive reasoning?

Answer: Yes. Bill and Jerry noticed a pattern; the first student loved the ice cream, the second student loved the ice cream, the third student loved the ice cream, etc. Bill and Jerry concluded that everyone would love the flavor, based on this pattern, and that it would be a great seller for the company. (If you ever take a class with Jerry or Bill, be sure to ask for the name of their newest flavor.)

Example 4: A coroner determines that a murder committed in Dallas occurred between midnight and 6 a.m. Frank, a key suspect in the investigation, boarded a flight from Hawaii to Dallas at 4 a.m. on the same day. Once the police verified this fact, Frank was no longer a suspect in the investigation. Is this an example of inductive reasoning?

Answer: No. In this case, the police deduced that Frank could not have committed the murder because he was either in Hawaii waiting for a plane or on the plane when the murder occurred. This type of reasoning is called deductive reasoning.

Using Inductive and Deductive Reasoning To Prove Conjectures

In the previous problems, we have shown how to use inductive reasoning to complete a pattern, as well as make a conjecture, but now we want to use it in tandem with deductive reasoning to prove a conjecture is true. Let's look at an example to see how they work together.

Example 5:

Part A: Choose any number. Multiply it by 10, and then add 40 to the product. Next, divide the sum by 5. Finally, subtract 8 from the quotient.

Let's look at several possible choices for your number, such as 3, 7, and 10. What happens when we repeat this set of steps for each number?

When $n = 3$, you get $3 \cdot 10 = 30$.	When $n = 7$, you get $7 \cdot 10 = 70$.	When $n = 10$, you get $10 \cdot 10 = 100$.
Now add 40: 30 + 40 = 70.	**Now add 40:** 70 + 40 = 110.	**Now add 40:** 100 + 40 = 140.
Divide by 5: 70 ÷ 5 = 14.	**Divide by 5:** 110 ÷ 5 = 22.	**Divide by 5:** 140 ÷ 5 = 28.
Finally, subtract 8: 14 - 8 = 6.	**Finally, subtract 8:**: 22 - 8= 14.	**Finally, subtract 8:** 28 - 8 = 20.
So when $n = 3$, you get 6.	So when $n = 7$, you get 14.	So when $n = 10$, you get 20.

Part B: Looking at the final answer in each situation, can you make a conjecture about the original number and the final result?

Answer: The final result seems to be two times the original, or $2n$. We have used inductive reasoning to make this conjecture; we have noticed a pattern and made a conjecture based on it.

Part C: Use deductive reasoning to prove your conjecture is true.

Now we need to use a little of our algebra knowledge to prove our conjecture is true. To do so, let's start with a general case by representing any number as n and then representing each of the steps noted in Part A with the proper mathematical symbols.

n	
$10n$	Multiply by 10.
$10n + 40$	Add 40 to the product.
$\frac{10n + 40}{5}$	Divide the sum by 5.
$\frac{10n + 40}{5} - 8$	Subtract 8 from the quotient.

Now that we have represented all the steps algebraically, we need to simplify. In order to simplify, we will factor out a 10 in the numerator and then reduce the fraction to lowest terms.

$\frac{\cancel{10}(n + 4)}{\cancel{5}} - 8$	Once the 10 is factored out, cancel the 5 into 10.
$2(n + 4) - 8$	Use the distributive property.
$2n + 8 - 8$	8 - 8 = 0
$2n$	

Answer: The result, $2n$, proves our conjecture is true—that following this set of steps will result in an answer that is twice the original number.

Practice Problems

Use inductive reasoning to determine the next number or symbol in the sequence.

1. 5, 9, 13, 17, __?
2. 7, 4, 1, -2, __?
3. 1, 5, 10, 16, __?
4. 27, 9, 3, 1, __?
5. 16, -4, 1, - ¼, ___?
6. 1, 4, 9, 16, ___?
7. @, @, #, $, @, @, #, ___?
8. △□, △△□, △△__□

Use inductive reasoning to test the conjectures that follow. If the conjecture is false, give at least one counterexample.

9. The product of two positive even numbers is always an even number.
10. The sum of any three odd numbers is always an odd number.
11. The difference between any two positive numbers is a positive number.
12. The quotient of two positive numbers is always greater than 1.

Use inductive and deductive reasoning in the following problems.

13. Choose any number. Multiply it by 4, and then add 8 to the product. Next, divide the sum by 2. Finally, subtract 4 from the quotient.
    a. State a conjecture about the result when performing this ordered procedure on any number.
    b. Prove your conjecture is true, using deductive reasoning.
14. Choose any number. Multiply it by 9, and then add 18 to the product. Next, divide the sum by 3. Finally, subtract 6 from the quotient.
    a. State a conjecture about the result when performing this ordered procedure on any number.
    b. Prove your conjecture is true, using deductive reasoning.

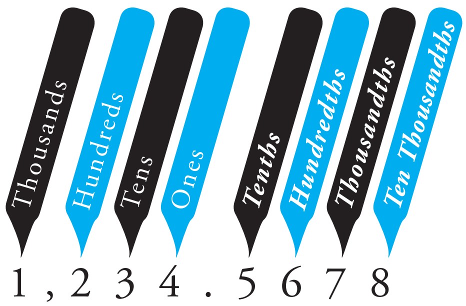
Section 2
Estimation

Introduction

Estimation is the process of finding an approximate answer to a numerical operation. It allows us to decide if an answer we calculated–whether by hand or with a calculator–seems reasonable. Estimation is quite important in the modern world; we tend to assume calculations made by machines are always correct, but sometimes the answers are wrong, often because of human error. If we are able to estimate, we are able to catch these miscalculations.

Rounding

Rounding a number consists of approximating it to within a certain multiple of 10. When rounding numbers, we always look at the digit to the immediate right of the digit to be rounded. Thus, if we are rounding to the hundreds place, we will look at the digit in the tens place, if we are rounding to the thousandths place, we will look at the ten-thousandths place, and so on. **If the digit to the immediate right is 5 or greater, we increase the digit to be rounded by one.** If not, we leave the digit to be rounded as is. In both cases, we replace all digits to the right of it with zeros. The following chart may assist you if you have forgotten the names of the place values for whole numbers and decimals.



Example 1: Round the number 51,896.3472 to each of the following places:

Part A: Thousandths—since the 7 occupies the thousandths place, look to the right of the 7 to see a 2. Since the value 2 is not greater than or equal to 5, the 7 will remain as is.

Answer: 51,896.347

Part B: Hundredths—since the 4 occupies the hundredths place, look to the right of the 4 to see a 7. Since 7 is greater than or equal to 5, round the 4 to the next integer, or 5.

Answer: 51,896.35

Part C: Ones. Since the 6 occupies the ones place, look to the right of the 6 to see a 3. Since the value 3 is not greater than or equal to 5, the 6 will remain as is.

Answer: 51,896

Part D: Tens. Since the 9 occupies the tens place, look to the right of the 9 to see a 6. Since the 6 is greater than or equal to 5, round 9 to the next integer, or 10. Write a 0 in the place of the 9, and then add 1 to the 8, which is in the hundredths place.

Answer: 51,900

Note: If you find this procedure confusing, try thinking of it this way: The two digits to the left of the 6 are 8 and 9. If you round 89 up, it becomes 90.

Example 2: Estimate each of the following, using the given directions.

Part A: Estimate by rounding each number to the nearest 100.

324 + 467 + 670 + 795

Before we can find the sum of these numbers, we need to round each of them to the nearest 100: 324 becomes 300, 467 becomes 500, 670 becomes 700, and 795 becomes 800. We add these rounded numbers: 300 + 500 + 700 + 800 = 2,300.

Answer: 2,300

Part B: Estimate the product by rounding each number to the nearest 10.

646
<u>x 52</u>

Before we can multiply, we round 646 to 650 and 52 to 50. We multiply these rounded numbers: 650 x 50 = 32,500.

Answer: 32,500

Note: Remember the following.

SUM means ***addition***

DIFFERENCE means ***subtraction***

PRODUCT means ***multiplication***

QUOTIENT means ***division***

Example 3: Estimate the following by rounding each number to the nearest tenth.

2.37 + .94 + 6.05 + 12.32

First round each number to the tenths, by looking at the hundredths place to see if it is 5 or greater. If it is, round up, if not, leave it alone. This gives us the following:

2.4 + .9 + 6.1 + 12.3

Now using a calculator or scratch paper, add the numbers.

Answer: 21.7

Applications of Rounding

Rounding numbers is the first step of estimating. Rounding allows you to work with numbers that are small enough to work with in your head. Practicing this type of estimation can be fun; you can estimate how much your grocery bill or lunch bill will be before you receive it. *(Okay, this mental activity is probably not the most fun you could ever have, but Bill and Jerry are math teachers; they lead very dull lives.)*

Estimating the Cost of Groceries

Example 4: You walk into a grocery store with a coupon for $5.00 off a purchase of $20.00 or more. You have $25.00 in your wallet, so you can buy about $30.00 worth of grocery items if you use the coupon ($30 – $5 coupon = $25). Below is the list of items that you need today. Estimate the cost of these grocery items by rounding to the nearest dollar (or ones place), to see if you will be able to purchase all the items with your $25.00.

Cereal: $2.79	Milk: $3.19	Cookies: $3.99	Cucumber: $.89
Bread: $2.09	Jelly: $4.69	Frozen Meal: $3.39	Ice Cream: $4.89

Since these grocery items are less than $10.00 each, a good habit is to round to the nearest dollar if you want to estimate. For this problem, each of the items is rounded below:

Cereal $3	Milk $3	Cookies $4	Cucumber $1
Bread $2	Jelly $5	Frozen Meal $3	Ice Cream $5

Once you add these dollar amounts together, you get an estimate of $26.00. Since you have the $5.00 coupon, you can afford this purchase.

Answer: You have enough money.

Estimating the Effect of a Raise on Your Paycheck

Example 5: Your boss just raised your hourly rate from $7.95 an hour to $8.82 an hour. For a 40-hour work week, approximately how much more will you make each week?

This problem is a common estimation problem. First, we round both hourly rates to the nearest dollar, or to $8.00 an hour and $9.00 an hour. Since the difference is about $1.00 per hour, we multiply 40 (hours) by $1, giving us the answer of $40.00. Thus, your new raise has garnered you about $40.00 extra (before taxes, of course) per week.

Answer: About $40.00 extra

Estimating the Distance You Can Travel

Example 6: If you can average about 70 mph for 2.8 hours of driving, about how many miles will you travel?

Solution: Before we can estimate this problem, we round the decimal to the nearest whole number; rounding gives us 3 instead of 2.8. Now we multiply 70 by 3, giving us an answer of about 210 miles. (Just remember that when you estimate in this way in the real world, you need to factor in the restroom breaks for those individuals with small bladders.)

Answer: 210 miles

Estimating the Amount of Food You Need for a Cookout

Example 7: Suppose you are having a cookout, and you are expecting about 12 adults and 18 children. For each adult, you want to make two burgers and for each child, just one burger. If each burger is going to be 1/4 of a pound in size, will a 5-pound package of ground beef be sufficient?

We personally relate to this dilemma, as Jerry likes to invite Bill and other members of the math department and their families to picnics. The first step is to determine how many burgers are needed. In this case, we are estimating two burgers per adult and one per child, giving us the following:

$$(12 \times 2) + (18 \times 1) = 24 + 18 = 42 \text{ burgers.}$$

From this point, we have at least two methods to approach the problem. The first would be to simply find the product of 42 and ¼ (or .25) to see how many pounds of meat are needed. This calculation tells us you would need 10.5 pounds, meaning you would need to more than double your purchase of ground beef.

A second method is to divide 5 pounds by .25 to see how many burgers could be made at this size. In this case, 5 divided by .25 is 20 burgers. You have to make 42, so either way you cut it (HA!), you will need more ground beef.

Answer: You will need more ground beef.

Finding a Car's Gas Mileage

When you buy a car, it usually is labeled with the number of miles per gallon it will get in highway driving and in city driving. An easy way to check the accuracy of this value is to keep track of the odometer reading before and after you fill up the car and also keep track of how much gas you put in the tank. Your MPG is then calculated using the following formula.

MPG = (ending odometer reading – starting odometer reading) / (gallons of gas used)

Example 8: John just bought a used car and was told by the salesman that its gas mileage was around 20 mpg in the city and 25 mpg on the highway. John goes immediately to a gas station to fill up his car and records the mileage on his odometer: 67,810 miles. He then proceeds to drive his car to and from work all week, making sure not to accelerate with a lead foot. At the end of the week, he fills up his car again, this time with 11.5 gallons of gas. He also notes the mileage on his odometer: 67,998 miles. How many miles per gallon did John's car get for this week of driving? Does this result support the salesman's claim?

In order to estimate the MPG of a vehicle, first we find the difference between the beginning odometer reading and the ending odometer reading. In this case, subtracting 67,810 from 67,998 is 188 miles. Since John filled up the car the second time with 11.5 gallons of gas, we divide 188 by 11.5, giving John's car an MPG of about 16.3. It looks like the salesman was not being honest with John.

Answer: About 16.3 mpg, which does not support the salesman's claim, since it is not in the range of 20–25 mpg as noted

Note: While it appears that we have an exact calculation of the miles per gallon for this car, remember that this mileage is only an estimate. One week John may experience more traffic, thus more starting and stopping, which would lead to a decrease in his gas mileage. And the next week could be a popular vacation week such as the week of Thanksgiving, so he would experience very little traffic at all, which would improve his gas mileage. Since his mileage varies from week to week, we should treat 16.3 as an estimate for that week of driving only.

Percents

Recall that a percent is simply another way of expressing a fraction or decimal. To change a decimal to a percent, multiply the decimal by 100 (move two decimal places to the right). To change a percent back to a decimal, divide by 100 (move two decimal places to the left).

A simple example of using percents is a purchase involving sales tax.

Example 9: Find the sales tax on each of the following items.

Part A: 8% sales tax on a car valued at $15,900

In order to find the sales tax on an item, change the percent to a decimal and multiply it by the cost of the item. Converted to a decimal, 8% is .08. Multiply: 15,900 x .08 = 1,272.

Answer: $1,272

Part B: 6.5% sales tax on a new cell phone listed at $249.99

First convert 6.5% to a decimal, which is .065. Then multiply: 249.99 x .065 = 16.24935. Since tax is always rounded to the nearest penny (the hundredths place), round the product to 16.25.

Answer: $16.25

Some other applications of percent are considered in the following examples.

Example 10: Suppose you want to purchase a house that costs $150,000.

Part A: Knowing that the bank will finance you for only 80% of the cost of the house, determine your required down payment.

If the bank will finance 80% of the cost of the house, you must put up 20% of the cost as your down payment: 20% of $150,000 = (.20) x (150000) = $30,000.

Answer: $30,000

Part B: If the terms of the loan stipulate payments of $681.35 a month for 30 years, determine what you ultimately will pay for this house if you don't make any extra payments.

You will pay $681.35 per month for 30 years. Thirty years is (12) x (30), or 360 months. Thus, you will ultimately pay (360) x (681.35), or $245,286 + the original down payment of $30,000, for a total of $275, 286 for this house.

Answer: $275,286

Note: Since the house was $150,000, and the cost to repay the loan was $275,286, this means you gave the bank $125,286 in interest charges to finance the house. Makes you wonder how banks ever lose money, doesn't it?

Example 11: Bill wants to buy a new computer for $1,600.00 (plus 7% sales tax). At checkout, due to his great credit, he is given two options. Using each option (below), determine what Bill would ultimately pay for the computer, and then determine the actual cost of his decision.

Option 1: Pay cash now for the computer to save 10% off the total cost of $1,600.00 (total cost does not include tax).

Paying 90% of the original price is equivalent to 10% off.

90% of 1600	Convert 90% to a decimal: .90.
0.90 x 1600	Multiply.

$1,440	Discounted cost—to determine the discounted cost plus the 7% sales tax, multiply by 1.07.
1440 x 1.07	
$1,540.80	Bill's ultimate cost for Option 1

Answer: $1,540.80

Option 2: Pay nothing down, and then make 24 equal payments of $85.00 (payments include tax).

24 x 85	
$2,040	Bill's ultimate cost for Option 2

Answer: Taking Option 1 saves Bill $2040 – $1540.80, or $499.20. (The installment plan almost always costs more money; don't take it if you don't have to.)

Calculating Percents Mentally (Optional)

Have you ever wanted to calculate the proper discount on sale merchandise or calculate the proper tip in a restaurant? Most people have no idea how to find these percents unless they have a calculator. The secret to calculating these percents is to use the 10% rule. Ten percent of any number is the number rewritten with the decimal moved to the left one place.

For example, 10% of $30.00 is $3.00, and 10% of $2.50 is .25. Notice the decimal was just moved one place to the left, and you have 10% of the number. Let's look at an example to see how to use this 10% rule.

Example 12: What is the standard tip on a restaurant bill of $50.00?

For the sake of this problem, we will assume the standard tip is 15%.

Step 1: Find 10% of $50.00.

Move the decimal one place to the left, and you have $5.00.

Step 2: How many 10%s make 5%? Well, half of 10% is 5%, so take half of $5.00.

Half of $5.00 is $2.50.

Step 3: Since 10% + 5% = 15%,

$5.00 + $2.50 = $7.50, which is 15% of the total bill. This is your tip.

Answer: $7.50

Note: In the real world, sale items and restaurant bills are almost never whole numbers, so it is common to round to the nearest whole number before trying to calculate anything mentally (for example, $35.59 would be rounded to $36.00).

Practice Problems

Round each of the following numbers to the following place values:

a. Hundredths b. Ones c. Thousands

1. 4,956.264
2. 50,997.3698

Estimate each of the following by rounding to the noted place value.

3. $54.15 + $32.98 + $1.34 (dollar)
4. .3 + .78 + .25 + 1.12 (tenths)
5. 40 x 2.95 (ones)
6. 43,287 x 1,059 (thousands)

Use mental percents to estimate each of the following.

7. 30% of $58.99
8. 15% of $32.00
9. 40% of $81.05
10. 70% of $88.99

Solve the following:

11. You walk into a grocery store with a coupon for $5.00 off the total cost if you spend more than $30.00. You have $35.00 in your wallet, so you can buy about $40.00 worth of grocery items if you use the coupon. To determine if you will be able to purchase all the following items with your $35.00, estimate the total cost by rounding.

Cereal: $2.89	Milk: $3.29	Cookies: $3.79	Olive Oil: $7.89
Bread: $2.19	Jelly: $4.89	Frozen Meal: $3.25	Ice Cream: $4.89

12. Your boss just raised your hourly rate from $11.05 an hour to $12.88 an hour. For a 40-hour work week, approximately how much more (gross) will you be making each week? If you are paid biweekly, what is the approximate difference (gross) you will see in your check?
13. If you can average about 60 mph for 3.9 hours of driving, about how many miles will you travel? If you have to travel about 350 miles, then approximately how many hours will your trip take at the rate of 60 mph?
14. Approximately how many 1/4-pound patties can be made from 8 pounds of ground beef? If you decided to make each patty 1/3 of a pound, how many fewer patties would be made from the same 8-pound package?
15. Kayla is trying to determine her miles per gallon on her new vehicle. She started with an odometer reading of 43,504 miles and ended with a reading of 43,836 miles after a fill up of 18.3 gallons. How many miles per gallon did the car get?

16. Suppose you want to purchase a house that costs $250,000. Knowing that the bank will finance you for only 80% of the cost of the house, determine your required down payment. Also, if the terms of the loan stipulate payments of $1,167.15 a month for 30 years, what will you ultimately pay for this house if you don't make any extra payments?
17. Alyssa has two options to buy a new fishing rod. Which option saves her the most money?
    a. Purchase the rod online for $90, no tax required, plus $11.99 shipping and handling.
    b. Go to a local store to buy it for $99.00 plus 7% tax.
18. Jerry wants to buy a new computer for $1,400.00 (plus 8% sales tax). At checkout, due to his great credit, he is given two options:
    a. Pay cash now for the computer to save 15% off the total cost of $1,400.00 (does not include tax).
    b. Pay nothing down, and then make 24 equal payments of $80.00 (includes tax).

    Using each option, determine what Jerry would ultimately pay for the computer, and then determine which option costs less.

Research and/or Discussion Questions

The following problems will lead you to think about the mathematics covered in this chapter on a little deeper level, as well as to discuss the real-world applications of this material. We want these problems to promote thought-provoking, research-based discussions either between you and your instructor and/or between you and your classmates. When discussing these problems, please be professional in your responses, keeping your emotions under control and remembering that the discussion is occurring in "just" a math class.

1. Profiling could be considered a form of inductive reasoning. Profiling is controversial, and your authors actually disagree on whether or not it is a valid technique. In your opinion, when should and shouldn't profiling be used, or should it never be used? If you believe profiling should be used, who should use it?
2. If you have ever served on a jury, you have probably experienced what it is like to deem someone guilty or not guilty. In your experience, did you find yourself more inclined to use inductive or deductive reasoning when making your decision? If you have never been on a jury, which reasoning process would you find most appropriate when deciding the fate of a defendant?
3. A few examples of inductive reasoning were noted in this text. What are some of the common, everyday instances of inductive reasoning that we hear or read about or observe? Does inductive reasoning always lead to the correct (an accurate) conclusion? (Hint: A recent example might be the idea that housing prices will always go up.)
4. Have you ever been presented with various payment options when purchasing an item, chosen an option after making a quick estimate, and then learned you made a mistake when estimating? When purchasing large-ticket items, sometimes we make quick estimates that can lead to financial distress or pleasure. Please share a situation in which you made such a decision, and comment on the results of this decision.

5. Tipping wait staff at a restaurant is considered customary. Over the past few years, 15%, 18%, and 20% have each been suggested as the "proper" tip. What are your feelings on the proper tip amount, and how do you justify leaving or not leaving a tip?
6. Have you ever estimated the actual cost of buying a home versus renting one or estimated the actual cost of buying a condo versus renting an apartment? Have you ever compared buying and renting to determine which way of living is actually cheaper? If you currently are purchasing a home, compare that cost to the cost of renting one in the same neighborhood, or vice versa. If you currently are renting an apartment, compare that cost to the cost of buying a condo with similar characteristics to your apartment, or vice versa. When you are finished with your estimates of renting versus owning, consider the tax implications or talk with your financial planner to see if you have overlooked anything. (So often we will surf the Internet to save $5 on a toaster, but we won't research our greatest expenses, our monthly house/apartment/condo payments.)
7. How important was gas mileage when you purchased or leased your car? Or how important was distance to your place of employment when you bought/rented your current dwelling? Have you ever determined the cost to you, per year, of those decisions? With gas prices ranging from $2.00 to $4.00 per gallon in the last few years, determine the following, using your car's estimated gas mileage:
    a. Cost savings per year to own a car that obtains 2 miles more per gallon than your current car obtains.
    b. The cost savings per year to live in a house 5 miles closer to your school or place of employment.

History Questions

The following topics have been given to you as ideas to research for history related to topics discussed in this chapter.

1. Reasoning: What is the history behind inductive and deductive reasoning? (Hint: Start with the Ancient Greeks; research Socrates and Plato.)
2. The place values: When were decimals first used? Did people resist their use?
3. The Fibonacci sequence: This sequence was presented to you earlier in this chapter, but the sequence itself has many applications. Research Fibonacci and his sequence, and find more applications. (Be careful: You will find online much nonsense about the Fibonacci sequence. You might enter *Fibonacci sequence flim flam* into your favorite search engine.)
4. Pascal's triangle: This triangle is another unique pattern of numbers like the Fibonacci sequence. Research Pascal's triangle, and look into any applications of this pattern of numbers.