CHAPTER 1

Evaluating Expressions and Formulas

Exponent Rules

Rule 1: To multiply identical bases, add the exponents.

Rule 2: To divide identical bases, subtract the exponents.

Rule 3: When there are two or more exponents and
multiply the exponents.
# Assignment Checklist

<table>
<thead>
<tr>
<th>What you should do:</th>
<th>Where?</th>
<th>When?</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Read the Introduction and Objectives</td>
<td>📚 or 📱</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Read all sections of the chapter and complete the Practice Problems</td>
<td>📚 or 📱</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complete the assignment and quiz (if applicable) in MathXL</td>
<td>📚 MathXL®</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Respond to the discussion question(s) for this chapter</td>
<td>📱</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other assignments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Notes:</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Learning Objectives

When you finish studying this chapter, you should be able to

• Simplify an algebraic expression by using the order of operations
• Evaluate a variable expression when given values for the variables
• Evaluate a formula when given values for the variables
• Solve word problems involving the simple interest formula

Order of Operations Agreement

In mathematics, algebraic expressions must be simplified in a prescribed order. Without this order, a single algebraic expression would have many possible values. For instance, the algebraic expression $10 - 2 \times 3$ would have two possible solutions: $24$ ($10 - 2 = 8$; then $8 \times 3 = 24$) or $4$ ($2 \times 3 = 6$; then $10 - 6 = 4$). In order to eliminate the potential problem of more than one possible solution, mathematicians use the following order of operations to simplify all algebraic expressions:

1: Perform operations inside grouping symbols first, and if there are grouping symbols nested within other grouping symbols, work from the inside out. Examples of grouping symbols are parentheses: ($$), brackets: [], and absolute value symbols: | |. If more than one operation is present in the grouping symbols, use the order noted in the following steps to simplify. For fractions, the numerator and denominator are treated independently.

2: Simplify exponential expressions or take roots in order from left to right.

3: Perform multiplication and division calculations in order from left to right.

4: Perform addition and subtraction calculations in order from left to right.

One of the most common errors in using the order of operations is to perform multiplication before division. This error is probably due to the instruction of the mnemonic PEMDAS, in which the M precedes the D, in middle or high school. Since M precedes D, students believe they should multiply before they divide. When this mnemonic was taught, the or between the M and D should have been emphasized as well as the or between the A and S. In other words, PEMDAS should have the following insertion.
Students often memorize PEMDAS as “Please Excuse My Dear Aunt Sally.” We recommend that you remember the following phrase:

**Please Eat Marshmallows or Doughnuts, Apples or Strawberries.**

This mnemonic emphasizes the word *or.*

*Note: Jerry and Bill can’t afford to pay for everyone’s dental work.*

Let’s now use the order of operations to simplify several problems.

**Example 1:** Simplify: \(2 - 16 ÷ 2 \times 2\)

Since there are no parentheses or exponents, we will perform the division first because it precedes multiplication when reading from left to right: \(-16 ÷ 2 = -8\). (Notice that we consider the subtraction sign a negative sign. In simplifying order of operations problems, *think of the sign to the left of a number as being connected to it.*)

\[
2 - 8 \times 2 \quad \text{Next, perform the multiplication: } -8 \times 2 = -16
\]

\[
2 - 16 \quad \text{Finally, subtract: } 2 - 16 = -14.
\]

\[-14\]

*Answer:* -14

**Example 2:** Simplify: \(7 - (8 - 3)\)

Since this problem contains parentheses, we must subtract 3 from 8 first.

\[
7 - (5) \quad \text{Finally, subtract: } 7 - 5 = 2.
\]

2

*Answer:* 2

**Example 3:** Simplify: \(6^2 - 4(2 + 7)\)

Since this problem contains parentheses, we must complete the operation inside them first before evaluating anything else: \(2 + 7 = 9\).

\[
6^2 - 4(9) \quad \text{Evaluate the exponent: } 6^2 = 6 \cdot 6 = 36.
\]
Multiply the 4 and the 9 next.

\[ 36 - 36 = 0 \] Subtract.

*Answer: 0*

**Example 4:** Simplify: \( 6 - 4^2 + 2 - 7 \)

The first step in this problem is to evaluate \( 4^2 \), which equals 16. Notice that when evaluating an exponent, we do **not** pay attention to the sign preceding the term unless it is in parentheses (see Example 27 in Review Chapter).

\[ 6 - 16 + 2 - 7 \] Then divide: \(-16 \div 2 = -8\).

\[ 6 - 8 - 7 \] Rewrite as an addition problem.

\[ 6 + (-8) + (-7) \] Combine the two negatives first.

\[ 6 + (-15) \] Subtract. Keep the sign on the 15.

*Answer: -9*

**Example 5:** Simplify: \( 5 - 3[24 \div (1 - 9)] \)

Whenever you have brackets and parentheses, you start with the innermost parentheses first and then work outward: \( 1 - 9 = -8 \).

\[ 5 - 3[24 \div (-8)] \] Now simplify \( 24 \div -8 \) in the brackets.

\[ 5 - 3[-3] \] Multiply -3 and -3 next. Don’t be lured into subtracting first.

\[ 5 + 9 \] Add.

14 *Answer: 14*
Example 6: Simplify: \( \frac{5 - 8 + 2}{4 + 3 \cdot 5} \)

Since this is a fraction, we treat the numerator and denominator independently. By choice, we will start with the numerator, which has absolute value symbols, so subtract 5 and 8.

\[
\frac{|-3| + 2}{4 + 3 \cdot 5} = \frac{3 + 2}{4 + 3 \cdot 5} = \frac{5}{4 + 3 \cdot 5} = \frac{5}{4 + 15} = \frac{5}{19}
\]

Answer: \( \frac{5}{19} \)

**Evaluating Variable Expressions**

Now that we have reviewed the order of operations, let’s try evaluating some variable expressions. To evaluate a variable expression, simply replace the variable with the given value, and then follow the order of operations. Always use parentheses to help avoid making sign mistakes when substituting the values in an algebraic expression.

Let’s consider a few examples.

Example 7: Evaluate \(5a - b\) when \(a = 2\) and \(b = -3\).

Rewrite the expression by replacing the variables with parentheses.

\[5(\_ \_ \_) - (\_ \_ \_)
\]

Substitute the values for \(a\) and \(b\).

\[5(2) - (-3)\]

Multiply the 5 and 2 first.

\[10 - (-3)\]

Subtract, and watch those signs!

\[10 + 3\]

Finally, add 10 and 3.
Example 8: Evaluate $x^2y^2 - 3xy + 1$ when $x = -3$ and $y = -1$.

Rewrite the expression by replacing the variables with parentheses, and keep the powers outside of the parentheses.

$$( )^2( )^2 - 3( ) ( ) + 1$$

Substitute the values into the parentheses.

$$( -3)^2(-1)^2 - 3(-3)(-1) + 1$$

Simplify the two powers $(-3)^2$ and $(-1)^2$. Then multiply:

$-3(-3) = +9$.

$(9)(1) + 9(-1) + 1$ Multiply 9 and 1, and then multiply 9 and -1.

$9 - 9 + 1$ Subtract 9 and 9.

$0 + 1$ Add.

$1$

Answer: $1$

Example 9: Evaluate $3(a - 2) + 5(7 - b)$ when $a = -1$ and $b = 5$.

Rewrite the expression by replacing the variables with parentheses.

$3( ( ) - 2 ) + 5( 7 - ( ))$

Substitute the values into the parentheses.

$3((-1) - 2) + 5(7 - (5))$ Drop the innermost parentheses around the $(-1)$ and $(5)$.

$3(-1 - 2) + 5(7 - 5)$ Simplify what is in the parentheses first.

$3(-3) + 5(2)$ Now multiply the 3 and -3, then the 5 and 2.

$-9 + 10$ Subtract 10 and 9.

$1$

Answer: $1$
Example 10: Evaluate $x - |y| + 2z$ when $x = -5$, $y = -3$, and $z = 7$.

First, we replace the variables with parentheses.

$$( - ) - | - | + 2 ( )$$

Notice that the absolute value sign is still present in the problem, and it is not replaced, since it is a grouping symbol. Now substitute the values for $x$, $y$, and $z$.

$$( - 5 ) - | - 3 | + 2 ( 7 )$$

Once values are substituted, perform operations within the parentheses and absolute value signs first. When the -3 leaves the absolute value signs, it becomes a 3. This part of the problem is not a “double negative” situation since you had to take the absolute value of -3 before you applied the subtraction sign.

$- 5 - 3 + 2 ( 7 )$  Now multiply the 2 and 7.

$- 5 - 3 + 14$  Finally, add and subtract in order from left to right.

6

Answer: 6

Evaluating a Formula

The formulas we use to find an unknown quantity in real life are simply algebraic expressions. Examples of real-life calculations include calculating the amount of interest paid on our savings accounts or calculating the perimeter of a backyard prior to installing a fence.

When we use a formula to find an unknown quantity, we are evaluating an algebraic expression by using the order of operations previously discussed in this chapter. In the following examples, we will use some of the basic formulas from geometry, as well as the simple interest formula.
**Basic Geometric Formulas**

### Areas

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>$A = \frac{1}{2}bh$</td>
<td><img src="image" alt="Triangle" /></td>
</tr>
<tr>
<td>Square</td>
<td>$A = s^2$</td>
<td><img src="image" alt="Square" /></td>
</tr>
<tr>
<td>Circle</td>
<td>$A = \pi r^2$</td>
<td><img src="image" alt="Circle" /></td>
</tr>
<tr>
<td>Rectangle</td>
<td>$A = lw$</td>
<td><img src="image" alt="Rectangle" /></td>
</tr>
</tbody>
</table>

### Perimeter

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>$P = s_1 + s_2 + s_3$</td>
<td><img src="image" alt="Triangle" /></td>
</tr>
<tr>
<td>Square</td>
<td>$P = 4s$</td>
<td><img src="image" alt="Square" /></td>
</tr>
<tr>
<td>Circle</td>
<td>$C = 2\pi r$ or $C = \pi d$</td>
<td><img src="image" alt="Circle" /></td>
</tr>
<tr>
<td>Rectangle</td>
<td>$P = 2l + 2w$</td>
<td><img src="image" alt="Rectangle" /></td>
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</table>

**Example 11:** Find the perimeter of a rectangle with length of 20 ft and width of 12 ft.

Since the perimeter of a rectangle can be found by using the formula $P = 2l + 2w$, we will simply substitute $l$ and $w$ into the formula and evaluate it.

$$P = 2(20) + 2(12)$$

Using the order of operations, we multiply 2 and 20, and then 2 and 12.

$$P = 40 + 24$$
Chapter 1: Evaluating Expressions and Formulas

\[ P = 64 \]

*Answer: \( P = 64 \text{ ft} \)

**Example 12:** Using the formula for the area of a triangle, find the total area of a triangle, given the base is 5 cm and the height is 8 cm.

\[ A = \frac{1}{2} \cdot b \cdot h \]

This is the formula for the area of a triangle. Substitute the values given, using parentheses around each substitution.

\[ A = \frac{1}{2} \cdot (5 \text{ cm})(8 \text{ cm}) \]

Multiply the numbers.

\[ A = 20 \text{ cm}^2 \]

All areas are measured in squared units: \((\text{cm}) \times (\text{cm}) = \text{cm}^2\).

*Answer: \( 20 \text{cm}^2 \)

**Other Basic Rate Formulas**

In the following table, we will introduce some basic “rate” formulas that we will use in solving some basic application problems for this course.

<table>
<thead>
<tr>
<th>Basic Rate Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Simple Interest Formula</strong></td>
</tr>
<tr>
<td>[ I = PRT, ]</td>
</tr>
<tr>
<td>where ( I ) represents interest, ( P ) represents the principal (the amount of money you started with or borrowed), ( R ) represents the rate (in decimal form), and ( T ) represents time borrowed or invested.</td>
</tr>
<tr>
<td><strong>Distance formula</strong></td>
</tr>
<tr>
<td>[ D = RT, ]</td>
</tr>
<tr>
<td>where ( D ) represents distance traveled, ( R ) represents the rate (or speed) traveled, and ( T ) represents the time traveled.</td>
</tr>
</tbody>
</table>

Let’s look at how these two formulas can be used in the following examples.
**Example 13:** Suppose that you borrowed $4,000 at the beginning of your pursuit of an Associate in Arts degree. After two years, you want to know the amount of simple interest that will be charged on the loan if the rate at which you are borrowing it is at 6%.

To solve this problem, we need to use the simple interest formula of \( I = PRT \). \( P \) is given at $4,000. \( R \) is given at 6%, which is .06 as a decimal, and \( T \) is 2 years. We will now substitute these values into the formula and solve for \( I \).

\[
I = \frac{PRT}{PRT} \\
I = 4000 \times 0.06 \times 2 \\
I = 240 \times 2 \\
I = 480
\]

The amount of simple interest that will be charged on the loan in 2 years is $480.

*Answer:*$480.00 in interest

**Example 14:** Suppose you are driving on the interstate at 65 mph for 4 hours straight. How far will you have traveled at the end of the 4 hours?

To solve this problem, we need to use the distance formula of \( D = RT \). \( R \) is the rate at which you are driving, which is given as 65 mph, and \( T \) is given as 4 hours. We will now substitute these values into the formula and solve for \( D \), the distance traveled.

\[
D = \frac{RT}{RT} \\
D = 65 \times 4 \\
D = 260
\]

260 represents the distance traveled.

*Answer: 260 miles

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**Practice Problems**

1. \( 9 - (2 - 6) \)
2. \( 18 - 6 + 3 - |{-2}| \)
3. \( 10 + (3 - 4 \cdot 2) \)
Chapter 1: Evaluating Expressions and Formulas

4. $3 \cdot (5 + 3)^2$
5. $6 + 4 \cdot (-3)$
6. $3 + 2 \cdot 5$
7. $18 \div 6 \cdot 4$
8. $6 + 2^3$
9. $5 - 6 \cdot 2^2$
10. $5 - (6 - 3) + 3^2$
11. $5[6 + 2(8 - 6)]$
12. $4[5 - 3(4 - 6)]$
13. $5 \cdot 3 - | -8 + 4 | \div 2$
14. $10 + (4 + 6 + 2) \cdot (-2)$
15. $6 - (-3 - 4 + 2)$
16. $5 + (-7 - 3(-2))^2 + |-5|$
17. $\frac{7 + 7(3 + 5)}{4^2 + 8}$
18. $\frac{|12 - 7| + 1}{3 + 4 \cdot 5}$

19. Simplify the following: $6 - 2 \times 4 - 10$
   a. 12
   b. -12
   c. 6
   d. -26

20. Simplify the following: $10 + 8 \div (-2) + 5 \times (-3)$
   a. -9
   b. 12
   c. -24
   d. -42

21. Simplify the following: $(2 - 6)^2 - 6$
   a. 58
   b. -14
   c. -20
   d. 10
22. Simplify the following: \(5 - (-2)^3 \div (-2)\)
   \[
   \begin{align*}
   a. & \quad 1 \quad & c. & \quad -6.5 \\
   b. & \quad 9 \quad & d. & \quad -5.5
   \end{align*}
   \]

For problems 23 through 30, evaluate each expression when \(x = 3\), \(y = -4\), and \(z = 6\).

   23. \(3x - 5\)
   24. \(y^2 - z^2\)
   25. \(3y^2\)
   26. \(2x + 4y\)
   27. \(|5z - 6x| + 5\)
   28. \(xy + yz\)
   29. \(y^3 + 3x^2\)
   30. \(3(x + y) - 7\)

For problems 31 and 32, \(a = -2\), \(b = -3\), and \(c = 5\).

   31. Evaluate: \(5a^2 - 4a + 3\)
   \[
   \begin{align*}
   a. & \quad 11 \quad & c. & \quad 31 \\
   b. & \quad -29 \quad & d. & \quad 20
   \end{align*}
   \]
   32. Evaluate: \(abc - b^2\)
   \[
   \begin{align*}
   a. & \quad 21 \quad & c. & \quad 39 \\
   b. & \quad -9 \quad & d. & \quad 9
   \end{align*}
   \]

33. Find the perimeter and the area of a rectangle with length 10 ft and width 15 ft.

34. Find the perimeter of an isosceles triangle with sides 12 ft, 15 ft, and 15 ft. (Note: An isosceles triangle has two sides with the same length.)

35. Find the perimeter of an equilateral triangle with sides 30 in. (Note: An equilateral triangle has three sides with the same length.)

36. Find the area of a triangle with base 30 ft and height 12 ft.

37. Find the perimeter and area of a square with sides 9 m.
38. Find the area of a circle with radius 10 ft. Use 3.14 for \( \pi \).

39. Find the amount of simple interest charged on a loan of $500 at a 3% rate for 5 years.

40. Find the amount of simple interest charged on a loan of $1,000 at a 3% rate for 2 years.

41. If you travel in a car at an average speed of 68 mph for 6 hours, how far will you have traveled? (Assume that this average speed factors in stops for gas and food purchases and bathroom breaks.)

42. If you travel in a car at an average speed of 60 mph for 7 hours, how far will you have traveled? (Assume that this average speed factors in stops for gas and food purchases and bathroom breaks.)